

Arshinikova Anasofia 354



Massachusetts Institute of Technology (MIT)



Lecture by Pr. Bob Gallagher
Boole (1815-1864) & Shannon (1916-2001)



Sapere aude!



Logical addition
(disjunction)

A	B	$A \vee B$
0	0	0
0	1	1
1	0	1
1	1	1

A	B	$A \vee B$
True	True	True
True	False	True
False	True	True
False	False	False



BOOLEAN LOGIC



+0.1
from
to
fin
Exas
Good logic



Socrates



Socrates



Plato



Aristotle



Socrates

Socrates was
a philosopher

philosophers are men

Socrates was
a man



$$\Phi \in A$$



$$S \in A$$



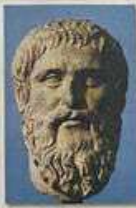
Bad logic



Socrates was
a man



Socrates



Plato



Aristotle

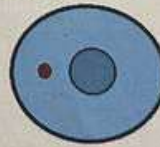


Socrates

philosophers are men

Socrates was
a philosopher

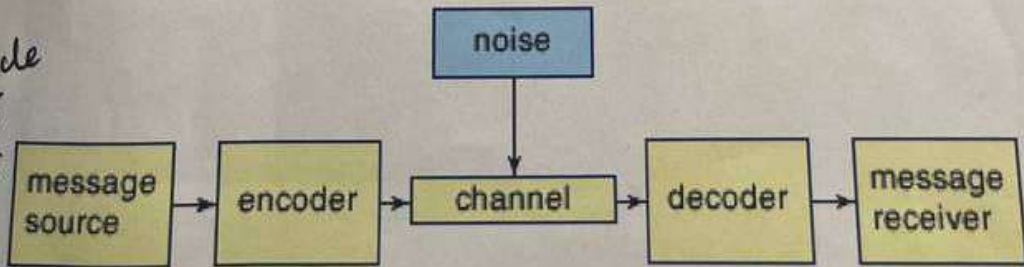
$$\Phi \in A$$



$$S \in \Phi$$



+0.1
to fin Grade
Feb 25
Handout



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(Shannon)

Resume of Lecture by Pr. Bob Gallagher from MIT

Massachusetts Institute of Technology (MIT)

George Boole (1815-1864) developed Boolean logic. The principles of logical thinking have been understood (and occasionally used) since the Hellenic era. Boole's contribution was to show how to systemize these principles and express them in equations (called Boolean logic or Boolean algebra).

Claude Shannon (1916-2001) showed how to use Boolean algebra as the basis for switching technology. This contribution systemized logical thinking for computer and communication systems, both for the design and programming of the systems and their applications.

Logic continues to be abused in politics, religion, and most non-scientific areas.

Logic continues to be abused in politics, religion and most non-scientific areas

(1)

Famous German figures



P. Kant Gauß M. Goethe W.

A little nationalistic, but this is an sample of right logic



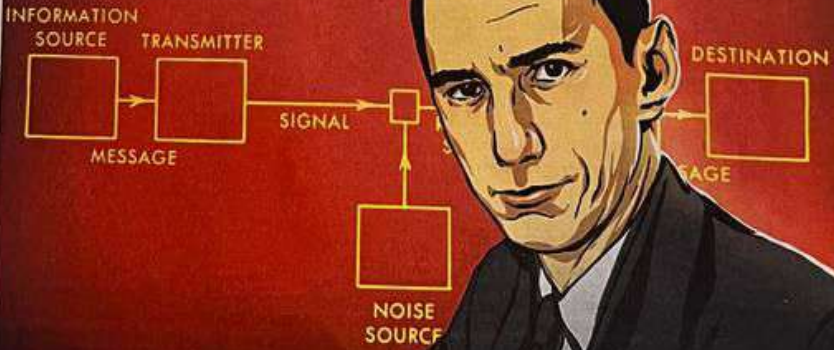
Arg. steps.



Bad logic (abuse of logic)

What did Shannon do while developing information theory? What is information theory about? (2)

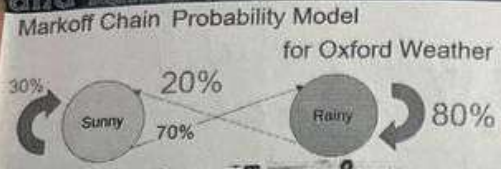
The Mathematical Theory of Communication



Creating a reliable connection over an unreliable (noisy) channel that's what IT is about

and that's what Shannon did

MIT Massachusetts Institute of Technology (MIT)
 Logic continues to be
 focused in politics,
 religion and most no
 scientific areas



What is this? (9)

School $\rightarrow$ gravity $\rightarrow$ MOTION $\Rightarrow$ formalism $\Rightarrow$ University $E = MC^2$ Φ Ψ $\dots$

Motivation: 80% chance of rain
 Let A_j be the event of rain at 9am on day j of this term, $1 \leq j \leq n$
 Suppose the events A_i each have probability p , independently

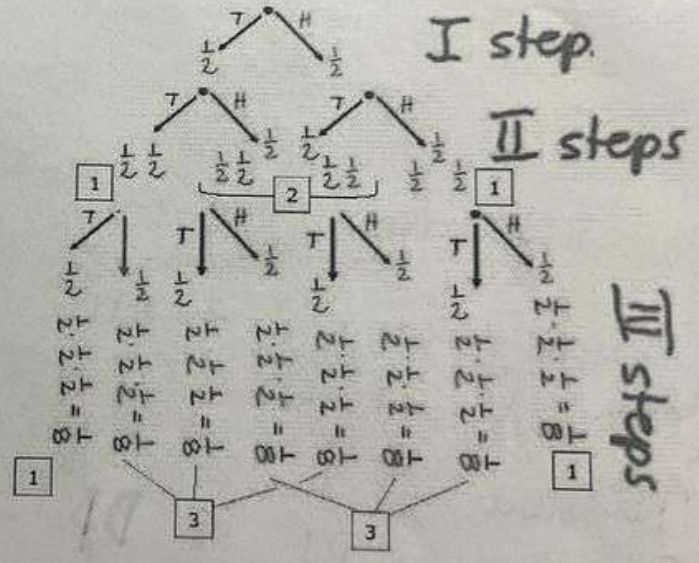
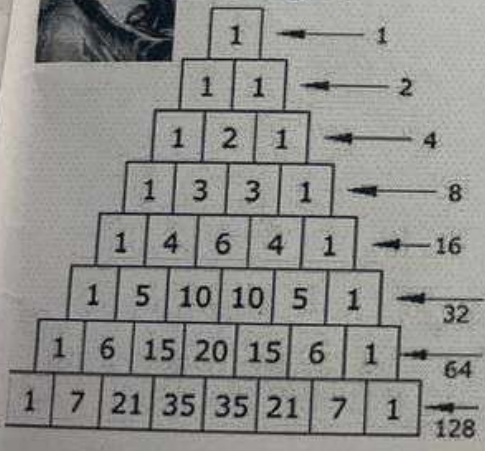
Oxford

Tue 13th	Wed 14th	Thu 15th	Fri 16th
10° 9°	13° 10°	13° 8°	11° 7°
70%	70%	70%	80%

(9)



Pascal's triangle



Newton's Binomial

$$\begin{aligned}
 (a+b)^0 &= 1 \\
 (a+b)^1 &= a+b \\
 (a+b)^2 &= a^2 + 2ab + b^2 \\
 (a+b)^3 &= a^3 + 3a^2b + 3ab^2 + b^3 \\
 (a+b)^4 &= a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4 \\
 (a+b)^5 &= a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5
 \end{aligned}$$



(7) A parity bit is a simple error-checking tool used in digital data transmission or storage. It's an extra bit added to a set of bits to check if the data is correct.

(8) The images - visual metaphors used in i.t., particularly about error correction and redundancy in common systems.

TOP IMAGE: a visual metaphor for redundancy. The character

Encoding the number 7.

7 In binary: $7 = 0111$

Position Bits: 1 2 3 4 5 6 7
? ? 0 ? 1 1 1

Positions: 1 2 3 4 5 6 7
Bits: P1 P2 ~~P3~~ P3 D2 D3 D4

Data bits 011 into positions 3(D1), 5(D2), 6(D3), 7(D4)

Then calculating parity bits P1, P2, P3

1. P1 (1357) → covers ? 0 1 1 → $0+1+1=2$ (even) → P1=0

2. P2 (2367) → covers ? 0 1 1 → $0+1+1=2$ → P2=0

3. P3 (4567) → covers ? 1 1 1 → $1+1+1=3$ (odd) → P3=1

So: Position: 1 2 3 4 5 6 7

Bits: 0 0 0 1 1 1 1

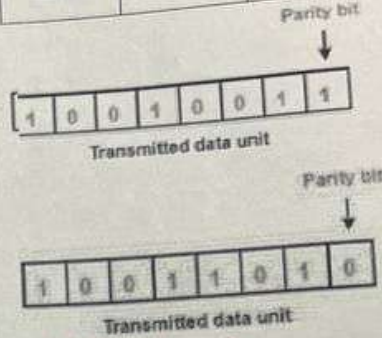
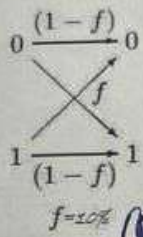
storage
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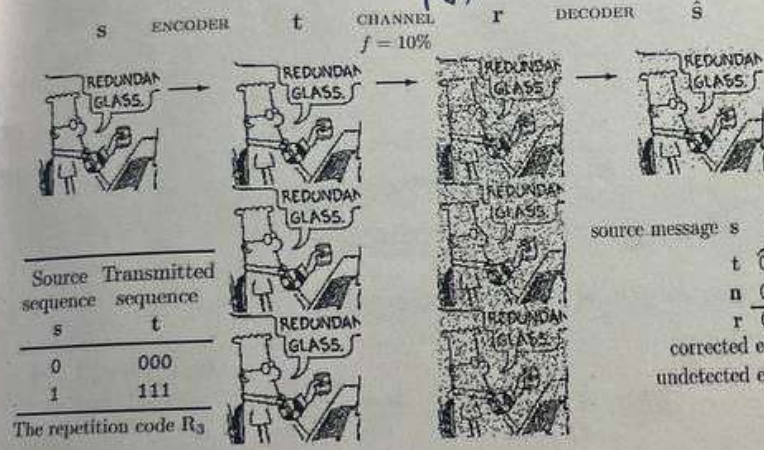
Sir Dr. D. MacKay,
University of Cambridge
(22 April 1967 - 14 April 2016)



"I believe in clean energy,
but I also believe in mathematics"



What is
Parity
Bit? (7)

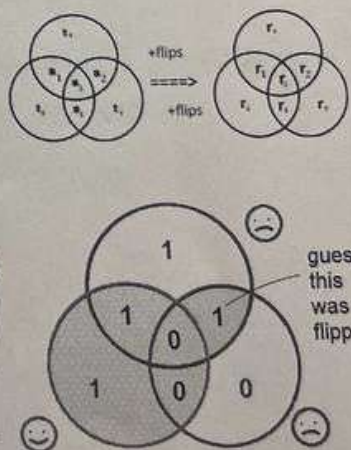
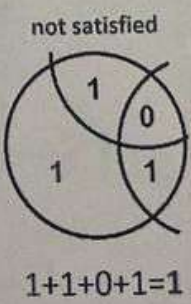
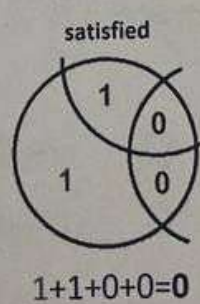
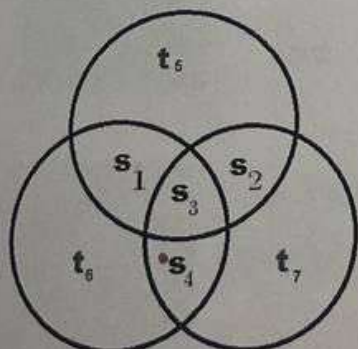


Source sequence	Transmitted sequence
s	t
0	000
1	111

The repetition code R_3

source message s	0	0	1	0	1	1	0
t	000	000	111	000	111	111	000
n	000	001	000	000	101	000	000
r	000	001	111	000	010	111	000
corrected errors	*						
undetected errors	*						

7.4. Hamming code. $\frac{4}{\Sigma} \rightarrow \frac{7}{t}$



Hamming 7.4 means: You take 4 data bits, add 3 ~~data~~ parity bits, and send 7 bits total

amplitude → Volume changes
AM (Amplitude Modulation)

mmmmmm

FM → Pitch (speed) changes
(Frequency Modulation)



Reginald A
(October 6, 186

"Ни одна орг
деятельност
области или
пока она не
University P

Battle of
(3 month 3
10.07-31.1



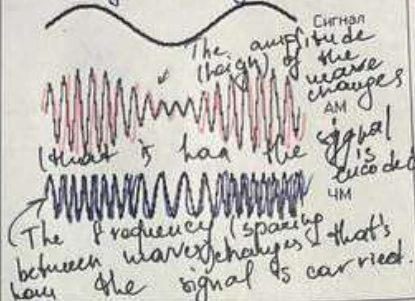
H. N

volume changes
speed changes

a comparison of AM and FM methods of encoding information into a radio wave.



Reginald A. Fessenden
(October 6, 1866 – July 22, 1932)

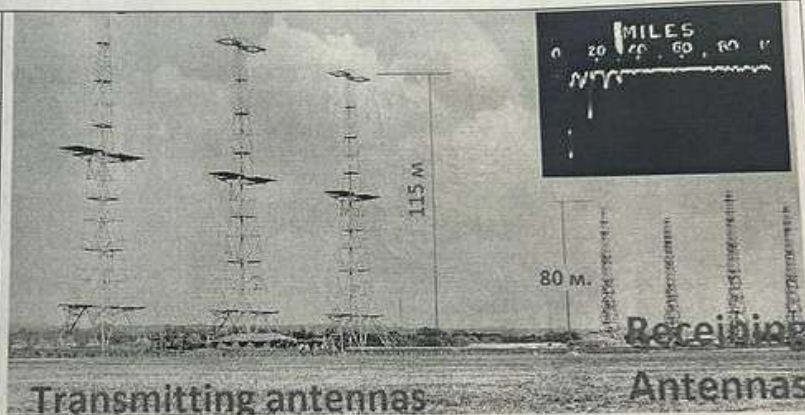


(October 6, 1866 – July 22, 1932)

first transmission of speech by radio (1900), and the first two-way radiotelegraphic communication across the Atlantic Ocean (1906)

"Ни одна организация, занимающаяся какой-либо конкретной областью деятельности, никогда не изобретает какие-либо важные разработки в этой области или не внедряет какие-либо важные разработки в этой области до тех пор, пока она не будет вынуждена сделать это из-за внешней конкуренции.." Oxford University Press. The Quarterly Journal of Economics, Feb., 1926, p. 262.

Battle of Britain
(3 month 3 weeks)
10.07-31.10.1940



Radar played a major role in the Battle of England

H. Nyquist



$$W = K \log m$$

Where W is the speed of transmission of intelligence,
 m is the number of current values,
and, K is a constant.



Ralph **Hartley** $H = n \log s$
(81:1888-1970)
 $= \log s^n.$

(u) What is Fourier transform?
It's a mathematical tool that turns signal from the time domain into the frequency domain

Example:



It's made of many notes (frequencies) played together

Fourier Transform breaks it down to show: Which notes are present
How strong each one is

Fourier Transform = from time

It helps us to see the ^{to frequency} hidden waves inside any signal - used in:

Music

Signal processing

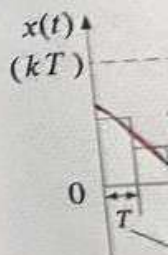
Image compression

Medicine (like MRI)

Fourier transform



Sampling



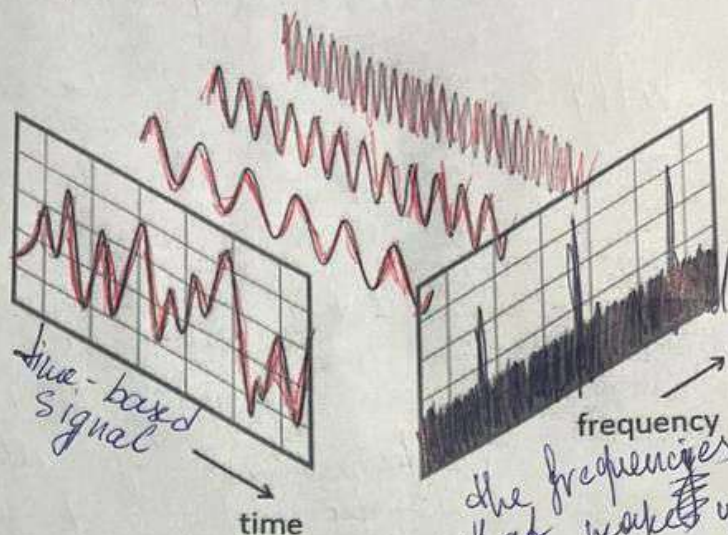
Time intervals (kT) are



$x(kT)$

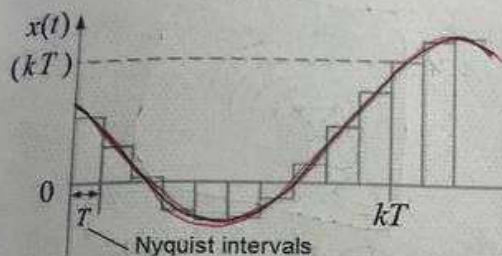
What is Fourier transform? (4)

Fourier transform



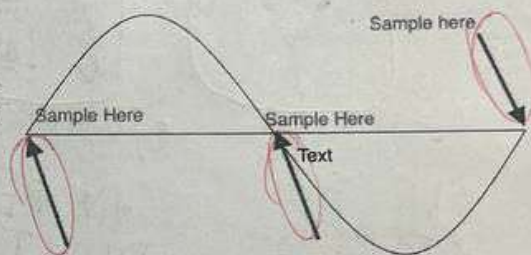
the frequencies that make up that wave

Sampling. Kotelnikov-Nyquist Theorem

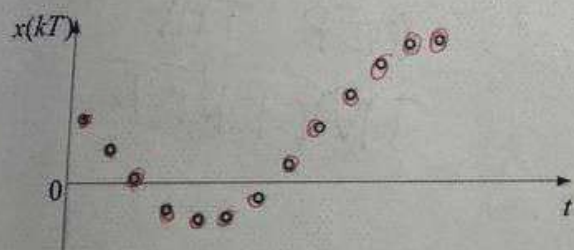


Sine with period T

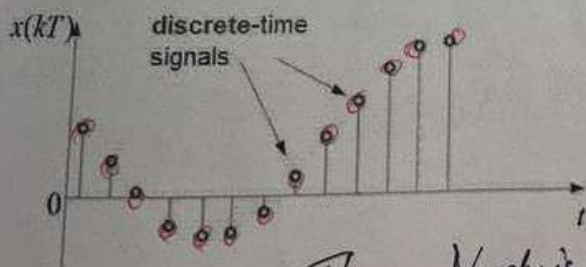
Sampling at $T/2$ (5)



Time intervals T , through which readings $s(kT)$ are taken, are called Nyquist intervals.



frequency Sample



$$F_{\text{sample}} \geq 2 * F_{\text{max}}$$

$$(T_{\text{sample}} \leq T_{\text{min}} / 2)$$

(6)

Vladimir
Kotelnikov

The Nyquist theorem tells us: to perfectly capture a signal, you must sample it at least twice the highest frequency in it

(5) Sampling - taking regular snapshots of a continuous signal to store or process in digitally
WHY? computers can't handle continuous data - so we "sample" it at regular time intervals

(6) Vladimir Kotelnikov independently discovered the Sampling theorem, which is foundation of all digital audio, video and communication
Thanks to him



• MUSIC CAN BE STORED AND STREAMED



• PHONE CALLS CAN BE DIGITIZED



• DATA CAN BE TRANSMITTED OVER THE INTERNET

shots of
sphere
confusion
at

Yes/No game of 20 questions



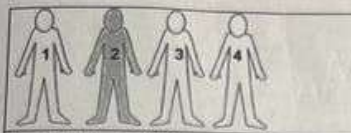
Say
NO to
the
first



Say YES to
the second
if it is better
than the
first



Say NO to the
third only if it is
worse than all
the others



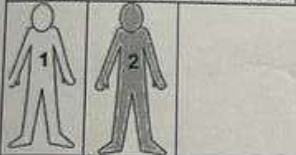
1 question - the first or second group



X

Question 1

2 question - the first or second group



X

Question 2

Average number of questions =

$$2 \cdot 0.25 + 2 \cdot 0.25 + 2 \cdot 0.25 + 2 \cdot 0.25 = 2$$

Average number of questions =

$$1 \cdot 0.5 + 2 \cdot 0.25 + 3 \cdot 0.125 + 3 \cdot 0.125$$



Question 1.
Is this
Zuckerberg?



50%

$$1 \cdot 0.5$$

Question 2.
Is this Sergey
Brin?



25%

$$2 \cdot 0.25$$

Question 3.
Is this Stefan
from BMW?



12.5%

$$3 \cdot 0.125$$

So Prince
Saud



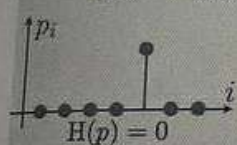
12.5%

$$3 \cdot 0.125$$

Average number of questions = 1.75

MITTE

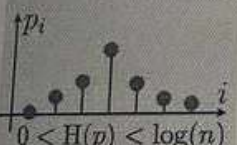
$$H(X) = - \sum_{i=1}^n p(x_i) \log_2 \frac{1}{p(x_i)}$$



$H(p) = 0$

$$\sum_{i=1}^n p(i) \log_2 \frac{1}{p(i)}$$

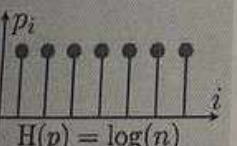
Quantifying
information



$0 < H(p) < \log(n)$

$$I(x_i) = \log_2 \left(\frac{1}{p_i} \right)$$

number of bits
required to encode
choice



$H(p) = \log(n)$

$$\sum_{i=1}^n p(x_i) I(x_i)$$

using the probabilistic grouping
and decision
trees to
minimize
entropy

50 billion 25 billion 12.5 billion 12.5 billion



Mark
Zuckerberg
 $P(1) =$
50%

Sergey
Brin
 $P(2) =$
25%

Stefan
Quandt
 $P(3) =$
12.5%

Prince Al
Saud
 $P(4) =$
12.5%

Mathematical expl. of the
Shannon Entropy

take a chance
about probability.

$$\begin{array}{r} A_1 B_1 B_2 A_2 - \\ A_1 A_2 B_1 B_2 \quad 4! \\ A_2 A_1 B_2 B_1 \quad 2! 2! \end{array}$$

$$B B B A T \rightarrow \frac{5!}{2! 2! 1! 1!}$$

$$T A L L I N = \frac{7!}{1! 1! 1! 1! 2! 2!}$$

$$\frac{120!}{5!} = \frac{6!}{3!}$$

6!

~~A A A B C D~~
distinguishable.
indistinguishable

$$\begin{array}{l} B A P A A_2 D \\ B A_1 A_2 A_3 C \rightarrow 3! \\ B A_1 A_2 A_3 C \end{array}$$

B A A B }
B B A A } 4

$B_1 B_2 A_1 A_2$
 $B_2 B_1 A_2 A_1$

$B_1 B_2 A_2 A_1$

$B_2 B_1 A_2 A_1$

$$\left. \begin{array}{l} -2 \\ -2 \end{array} \right\} 4$$

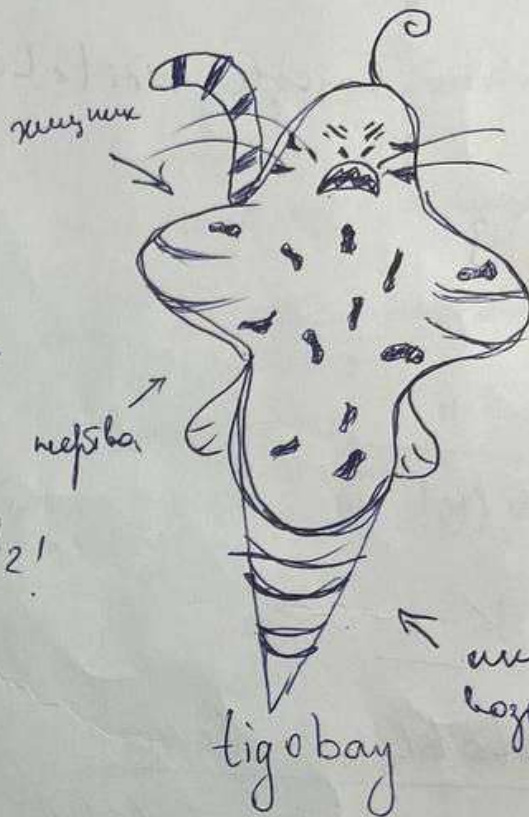
$$\frac{4}{4!}$$

$$n! = 1 \cdot (n-1) \cdot (n-2) \cdots 2 \cdot 1$$

$$P(A) = \frac{\text{ways considering } M_1! \cdot M_2!}{n!} \leftarrow n - m_1$$

$$M_1 + M_2 = n$$

ребячество



мужик

тигубай

мужик
богатырь

using System;

class ABoole

{
public static void Main()

{
Console.WriteLine("Hello Projecto")

}

}



$$\frac{n!}{m_1! m_2! \dots m_k!}$$

$k-2 \rightarrow$

Aristotelian Logic

- Who: Aristotle
- Based on: Categories (All, some, none)
- Examples
 - All humans are mortal
 - Socrates is human
 - Socrates is mortal
- Used in: Philosophy, reasoning

Boolean Logic

- Who: George Boole
- Based on: True/False (1/0)
- Uses: AND, OR, NOT
- Example:
 - A = true, B = false
 - A AND B = false
- Used in: Computer and electronics

parity bit -

англ. no capture

$H = n \text{ bits} = \log_2 n$

шифрование no capture
кодирования

(2.) Shannon created a mathematical framework that describes how information is transmitted through a communications system.

He: • Broke down communication into key components:

1. Information source
2. Transmitter
3. Channel
4. Noise source
5. Receiver
6. Destination

• Analyzed how information can be ~~reliably~~ encoded, sent and decoded reliably even in the presence of noise.

Difference between them two!

- A: Words and logic in statements
B: Math-like with true/false values

Замесилера думатора
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сбросователю или
наповазном Умелом
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ционными ресурсами"
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получение высшего
образования
Александров Александр
Максимович
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tigobay

Обязательно надо



catobee



исходно
рам



50%
 ♠ ♠
 + +
 ♥ ♥
 Card Deck
 50%
 1. Red? 2 questions
 No
 2) + No

♠
 1) + + + + $\frac{4}{8}$ 50%
 $\frac{4}{8} = \frac{1}{2} = 25\%$

+ $\frac{1}{8} = \frac{1}{8} = 0.125$ 12.5%
 ♥ $\frac{1}{8} = \frac{1}{8} = 0.125$ 12.5%

1) + $1 \cdot 0.5 = 1 \cdot \frac{4}{8} = 0.5$
 2) ♠ $2 \cdot 0.25 = \frac{2}{4} = 0.5$
 3) ★ $3 \cdot 0.125 = 3 \cdot \frac{1}{8} = 0.375$
 3) ♥ $3 \cdot 0.125 = 3 \cdot \frac{1}{8} = 0.375$
 1.75

$\frac{1}{8} + \frac{2}{8} + \frac{3}{8} = \frac{4+4+6}{8} = \frac{14}{8} = 1.75$

14 18
 8 1.75
 60
 56
 40

0.375
 0.375
 1.750
 5
 0.625

+ + + + + $\frac{5}{8}$
 ♠ $\frac{1}{8}$
 ★ $\frac{1}{8}$
 ♥ $\frac{1}{8}$

1) $1 \cdot \frac{5}{8} = 0.625$
 2) $2 \cdot \frac{1}{8} = 0.25$
 3) $3 \cdot \frac{1}{8} = 0.375$
 3) $3 \cdot \frac{1}{8} = 0.375$
 = 1.625

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(1) may

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2. P

3. P

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(3)

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2

If

religious ~~concepts~~

(1) "in politics"

may be abused when:

1. Politicians use misleading arguments to persuade or manipulate
2. Populist rhetoric replaced debate
3. Decisions are made for emotional or ideological reasons, ignoring facts

"in religion"

1. may be misused or distorted when:

1. Beliefs are asserted without evidence and then defended through circular reasoning
2. Faith is used to dismiss logical inconsistencies or avoid scrutiny
3. Appeals are made to tradition or authority instead of reason

"most non-scientific areas"

1. Pseudoscience

2. Conspiracy theories

3. Marketing and adv.

4. Social media arg.

(3) Image shows a Markov Chain which is a mathematical model used to describe how something changes from one stage to another based on probabilities

2 Weather states: Sunny ☀️

Rainy ☁️

If it's Sunny

70% chance it stays Sunny
30% it is going to be rainy tomorrow

If it's Rainy

80% ch. it stays Rainy
20% ch. it becomes Sunny

Key idea of M.C.

It assumes that the next state only depends on the current state, not the whole history. This is called

Marker Property

(9) What
Pascal's
numbers

Is Pascal triangle?
Triangle is a triangle of
where:

- Each number is the sum of the two numbers directly above it
- The triangle starts with 1 at the top.

Why created?

To help with math problems like:

- Counting things
- Solving equations
- Figuring out chances

Difference between information and data

Data are raw facts and figures without context, while information is processed data that has meaning and usefulness.

Subject of research in information theory

Information theory studies the quantitative characteristics of information and the methods of its transmission, storage, and processing.

Markov chains

Markov chains are mathematical models that describe systems comprising states and transitions, where the probability of the next state depends only on the current state.

Control codes

Control codes are mechanisms used to detect and correct errors in data during transmission or storage.

 $\mathbb{R}^3$

$\mathbb{R}^3$ denotes the space of three-dimensional vectors with real coordinates represented as $\mathbb{R}^3$.

Hamming code

Hamming code is an error-correcting method that adds redundancy bits to the original data for protection against errors.

Amount of information on Hartley

The amount of information according to Hartley is measured in bans and is defined as the logarithm of the number of possible messages.

Amount of information on Shannon

Shannon's amount of information considers the probabilities of messages and is measured in bits, reflecting uncertainty.

Concept of entropy

Entropy is a measure of uncertainty or randomness of a random variable, indicating the average amount of information.

Hartley definition of entropy

Hartley entropy is defined as the logarithm of the number of possible states of a system.

Shannon entropy

Shannon entropy generalizes Hartley's definition, taking into account the probabilities of different states.

Entropic compression of information

Entropic compression is the process of reducing the volume of data without loss of information, based on statistical properties of the data.

Probabilistic way of measuring information

This method measures information based on the probability of occurrence of an event, such as through entropy.

Nyquist interval

The Nyquist interval defines the maximum frequency that can be captured without aliasing for signal transmission.

Fourier transform

The Fourier transform converts time-domain signals into frequency representation, allowing analysis of frequency components.

Kotelnikov-Nyquist theorem

This theorem states that to reconstruct a signal, it must be sampled at least at twice the maximum frequency of the signal.

Pascal's Triangle

Pascal's Triangle is a tabular representation of binomial coefficients, used in combinatorics.

Binomial distribution

The binomial distribution describes the number of successes in a series of independent trials with two possible outcomes.

Boolean algebra and Aristotelian logic

Boolean algebra is a system for working with truth values, while Aristotelian logic focuses on rules of inference and syllogism.